Question Difficulty – How to Estimate Without Norming, How to Use for Automated Grading

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Abstract

Question difficulty estimates guide test creation, but are too costly for small-scale testing. We empirically verify that Bloom’s Taxonomy, a standard tool for difficulty estimation during question creation, reliably predicts question difficulty observed after testing in a short-answer corpus. We also find that difficulty can be approximated by the amount of variation in student answers, which can be computed before grading.

We show that question difficulty and its approximations are useful for automated grading, allowing us to identify the optimal feature set for grading each question even in an unseen-question setting.

1 Introduction

Testing is a core component of teaching, and many tasks in NLP for education are concerned with creating good questions and correctly grading the answers. We look at how to estimate question difficulty from question wording as a link between the two tasks.

From a test creation point of view, knowing question difficulty levels is imperative: Too many easy questions, and the test will be unable to distinguish between the more able test-takers, who all achieve equally good results. Too many hard questions, and only the most able test-takers will be clearly distinguishable from the (low-performing) rest.

In large-scale testing, question difficulty and other measures of question quality are established through prior norming (Downey, 2010), where the questions are answered by a pool of test-takers in a dry run before definitive use with a similar demographic. Difficulty is then determined on the basis of the observed results using probabilistic test theory (PTT). Norming is usually not available in automated question creation or in ad-hoc testing in small classrooms, while the need for correctly determining question difficulty of course remains.

In this situation, teachers often use Bloom’s Taxonomy (Bloom, 1956), a classification of the knowledge dimensions and cognitive processes involved in the completion of a test task, to formulate questions of appropriate difficulty. In the literature, the difficulty of multiple-choice questions has been successfully aligned with the cognitive process dimension of the Bloom hierarchy (Tiemeier et al. (2011); Kim et al. (2012), but see also Kibble and Johnson (2011)). In this paper, we empirically evaluate the predictive power of both Bloom dimensions for estimating the empirically observed difficulty of short-answer questions, which require the test-taker to freely formulate one to three sentence answers. We find that the Taxonomy allows a useful approximation of question difficulty at the time of question creation. We find clear empirical evidence that the instructional context, that is the teaching materials presented in instruction, has to be taken into account when determining difficulty using the Taxonomy.

Once test-taker answers are available, but before grading makes PTT analysis possible, another predictor for question difficulty becomes available: Answer variation, the average amount of variation within the student answers for each question, is computed based only on the answer strings.

We also look at question difficulty from the point of view of improving automated short-answer grading (SAG). To date, the focus of research has been on finding informative features, ranging from deep processing (Zesch et al., 2013; Hahn and Meurers, 2012) through text-based similarity (Sultan et al., 2016) to shallow, string-based approaches (Okoye et al., 2013; Jimenez et al., 2013). Padó (2016) has proposed to perform pre-grading model selection by tailoring feature sets to the characteristics of
different short-answer corpora. We refine this idea and show that within the same corpus, questions with different difficulty levels also profit from different feature sets, and that the Bloom Taxonomy levels and student answer variation can be used as stand-ins for feature set prediction if difficulty estimates are not available. These results point to a new avenue of research in SAG.

The paper is structured as follows: We begin by providing some theoretical background on PTT and Bloom’s Taxonomy in Section 2. Our first set of analyses tests the reliability of the Bloom’s Taxonomy question difficulty predictions for our data set (Section 3). The second analysis in Section 4 focuses on the relationship between answer variation and question difficulty. Our final set of experiments investigates the use of question difficulty for question-level model selection in short-answer grading (Section 5). We end with a discussion and conclusions in Section 6.

2 Theoretical Background

Our analyses require defining ground truth question difficulty. We use the Rasch model from probabilistic test theory for this estimate. This Section also introduces Bloom’s Taxonomy, a tool from the field of education intended for analysing the cognitive requirements for answering a question, and thereby its difficulty.

2.1 PTT Difficulty Estimation with the Rasch Model

Test theory is concerned with determining test-taker ability and analysing question quality and difficulty. Probabilistic test theory formulates latent trait models for these tasks. Latent trait models assume that a student’s ability and a question’s difficulty are not directly observable, but depend probabilistically on the observed scores. The two best-known proponents are the Rasch model (Rasch, 1960) and the related Item Response Theory models (van der Linden, 2010).

The Rasch model fits a joint model of question difficulty and student ability on the basis of the manual grades awarded to student answers (i.e., after testing). The goal is to establish question difficulty independently of concrete test-takers and vice versa. Concretely, the Rasch model estimates question difficulty and student ability given the following relation (where $B_n$ is the ability of student $n$ and $D_i$ is the difficulty of question $i$):

$$P_{ni}(x=1|B_n,D_i) = \frac{e^{(B_n-D_i)}}{1 + e^{(B_n-D_i)}} \quad (1)$$

Success ($x=1$) of a student $n$ on a question $i$ is linked to the difference between the student’s ability and the question’s difficulty. If the ability is greater than the difficulty, the student is likely to succeed, or if the inverse is true, the student is more likely to fail. Estimates of $B$ and $D$ are made iteratively from the test results.

The resulting measures are returned in logits and question difficulty is centered at 0, so that easy items have low or negative difficulty estimates and hard items have high difficulty estimates.

2.2 Bloom’s Taxonomy

Bloom’s Taxonomy (Bloom, 1956), revised by Anderson and Krathwohl (2014), is a well-known tool for creating and interpreting teaching objectives as well as writing test questions and estimating their difficulty. The Taxonomy has two independent dimensions: the Cognitive Process (CP) dimension and the Knowledge dimension (KD). The Cognitive Process dimension describes which type of cognitive activity is necessary to complete a task, in our case to answer a question. The least demanding process is Remember, followed by Understand (e.g., explain, compare, classify), Apply, Analyze and, the most demanding, Create.

The second dimension of the revised Taxonomy looks at the type of knowledge needed to complete the task. The simplest knowledge type is Factual (facts and terminology), followed by Conceptual (categories, principles and models), Procedural (algorithms, techniques and criteria) and Metacognitive (including strategic knowledge and self-knowledge).

Anderson et al. explicitly recommend that Taxonomy users infer the dimension levels from the question wording: Verbs like “compare” or “generalize” indicate the Understand level, while “identify” or most simply “name” belong to the Remember level. To assess the Knowledge dimension level needed to solve a task, Anderson et al. advise teachers to look at the direct object of the verb describing the required Cognitive Process. This explicit operationalization of level identification as analysis of the question formulation indicates the possibility of automating the process. Making these inferences however is complex for questions which set
**Table 1:** Example question with Bloom categories (original and re-assigned, see Section 3.3) and Rasch difficulty (centered at 0, larger means harder)

<table>
<thead>
<tr>
<th>Question</th>
<th>Warum hat jede Klasse die Methode public String toString()?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference Answer</td>
<td>Die Methode wird von der Klasse Object an alle Klassen in Java vererbt. The method is inherited by all Java classes from class Object.</td>
</tr>
</tbody>
</table>

concrete tasks. For example, for the question “Calculate the voltage given I and R.”, we need to infer that Ohm’s law, a generalization, will be applied to a concrete problem to arrive at the Apply cognitive process on the Conceptual level.

### 3 Bloom’s Taxonomy and Difficulty

We now empirically evaluate how accurately Bloom’s Taxonomy (Bloom, 1956), revised by Anderson and Krathwohl (2014), predicts question difficulty as estimated from student performance in a manually graded short-answer corpus. We check whether questions on the different levels of the Taxonomy show different ground-truth difficulty, as provided by a Rasch model.

#### 3.1 Data

We use the Computer Science Short Answers in German corpus (CSSAG, Padó and Kiefer (2015)). This corpus contains 31 content-assessment questions with reference answers as well as student answers by highly-proficient speakers of German (native or near-native). Anonymized student IDs are available to track answers by the same person, and there is sufficient person overlap between the questions to allow consistent PTT analysis.

We exclude question 6 from our data set. Rasch modelling uncovered an extreme mismatch of expected and actual difficulty, and further inspection of the answers shows that the question was often misunderstood and therefore skipped or answered incorrectly. Uncovering questions like this is one of the standard uses of PTT, so we feel justified in excluding the question after careful analysis.

#### 3.2 Method

For the empirical evaluation of Bloom’s Taxonomy levels, we annotated the CSSAG questions with the corresponding Cognitive Process and Knowledge dimension. The author’s annotations were verified by comparison to the level annotations of two colleagues familiar with the Taxonomy and the CSSAG subject matter, A and B.

The Cognitive Process annotations show substantial annotator agreement (UP-A: $\kappa = 73.7$; UP-B: $\kappa = 82.6$; A-B: $\kappa = 67.5$). Literature results, which mostly consider multiple choice questions, are often not this robust (Kibble and Johnson (2011): $\kappa = 33.3$, Cunnington et al. (1996): at most $\kappa = 48$ for a binary decision).\(^2\)

The Knowledge dimension is much less consistent (UP-A: $\kappa = 11.8$; UP-B: $\kappa = 24.9$; A-B: $\kappa = 32.8$). Analysis showed that the annotators entertained substantially different interpretations of the levels, making adjudication impossible. Classifying the Knowledge levels involves the annotators’ private conceptualisations of the question topic domain (What comprises procedural knowledge in Computer Science?), which leads to much greater inconsistency than classifying the process verb for the CP levels.

We use the author’s level annotations, with the caveat that the Knowledge level annotations are noisy. We found questions on the Remember ($n = 10$), Understand ($n = 17$) and Apply ($n = 3$) levels of the CP dimension and in the Factual ($n = 10$), Conceptual ($n = 18$) and Procedural ($n = 2$) levels of the Knowledge dimension.

We also estimated question difficulties on the basis of the student performance in the corpus. Table 1 shows a question from CSSAG with its reference answer and Bloom levels as well as its estimated difficulty.

Since we are doing data analysis and not building predictive models, we used the whole corpus without holding out test data.

For our analyses, we use the `lm` function in R\(^3\) to induce linear models for question difficulty, us-

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\(^2\)Kim et al. (2012) argue that level assignment is harder for multiple choice questions because the answer choices may provide clues to the students, effectively reducing higher-level questions to Remember.

\(^3\)www.r-project.org
### 3.3 Analysis I: Bloom’s Cognitive Processes and Difficulty

We begin by analysing the relationship between Bloom’s CP dimension and ground-truth difficulty. We train a linear model of difficulty, using the three CP levels present in the data as factors. However, the linear model is not significant, and neither are the coefficients. From this first analysis, it seems that Bloom’s Cognitive Process dimension cannot predict observed question difficulties.

A closer look at the Taxonomy description reveals a problem. The Cognitive Process dimension was first annotated taking only the question into account. However, Anderson and Krathwohl (2014) (p.71) state that “If the assessment task is identical to a task or example used during instruction, we are probably assessing remembering, despite our efforts to the contrary.” It is quite intuitive that, beyond the specific wording of the question, instructional context influences question difficulty. Therefore, we analysed the teaching materials (lecture slides) used for instruction before the CSSAG questions were answered in a test. The categories were then re-assigned with the teaching materials in mind: If for an Understand question, there was text presented on a single slide (or on several slides for a multi-component question) that would have been graded as a correct answer given the reference answer, the question was classified as Remember instead, since no active knowledge transfer was required by the student in this case. We re-classified six of originally 17 Understand questions as Remember (among them the example question in Table 1). The new classification based on this closer reading of Anderson et al. is called Bloom’s CP text&question levels below.

Table 2 shows the results of another linear model of ground-truth difficulty using the three CP text&question levels as factors. The model is significant on the $p < 0.05$ level, so the use of instructional context yields a quantifiable relationship between the Bloom levels and ground-truth difficulty. This relationship is carried by the Understand level - this model coefficient is significant and positive, meaning that Understand questions are predicted to have higher than average difficulty. The non-significant negative coefficient for Remember indicates a tendency for these questions to be less difficult than average. The estimate for the Apply level is based on only three data points, so the strong tendency for easier-than-average difficulty must be taken with a grain of salt. Unlike the findings for Remember and Understand, this last observation is not in line with the predictions of the Bloom Taxonomy. We return to this in Section 3.5.

In sum, the categories do show a significant difference in difficulty, but only if the explicit presentation of material during instruction is considered. The Bloom CP text&question categories are by design strongly correlated with the existence of the answer in the teaching materials: Questions in category Remember always refer to explicitly presented material, while questions in category Understand never do. Therefore, the predictive performance of the CP question&text levels could in principle be due just to the existence of the answer in the teaching materials. We therefore trained a linear model of difficulty using answer presented (1 if the answer was shown on the lecture slides, as defined for the category re-assignment above, 0 otherwise) as a factor. This model did not reach significance. We conclude that the predictive power of the Bloom dimensions (when assigned with the teaching materials in mind) is in fact at the core of our findings.

### 3.4 Analysis II: Bloom’s Knowledge Dimension and Difficulty

We now turn to the Knowledge dimension of Bloom’s Taxonomy. In the data, we find 10 questions on the Factual level, 18 on the Conceptual level and two on the Procedural level. The KD levels are not related to the answer presented measure: While answering a question may require knowledge that has been explicitly presented, the correct

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4In category Apply, explicitly presented or inferrable facts have to be applied to a new situation, so there is no a priori relationship between the category and the answer having been presented. In our data, all Apply questions referred to presented material.
Table 3: Difficulty and the Knowledge levels: Linear model coefficients. **: $p < 0.01$, ns: not significant.

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Std. Error</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>KD Factual</td>
<td>-0.785</td>
<td>0.263</td>
<td>**</td>
</tr>
<tr>
<td>KD Conceptual</td>
<td>0.316</td>
<td>0.196</td>
<td>ns</td>
</tr>
<tr>
<td>KD Procedural</td>
<td>0.280</td>
<td>0.588</td>
<td>ns</td>
</tr>
</tbody>
</table>

answer need not have been.

Table 3 shows the coefficients of another linear model of difficulty, now using the Knowledge dimension levels and again fixing the intercept at 0. The model predictions are significantly correlated with difficulty ($p < 0.05$). The significant coefficient is Factual knowledge, which results in the prediction of easier-than-average difficulty. This, of course, agrees with the Bloom Taxonomy.

Despite the large disagreement between the three annotators on this dimension, the annotated Knowledge levels still hold relevant information with regard to question difficulty, and that information is in line with the predictions of the Bloom Taxonomy.

3.5 Analysis III: Both Bloom Dimensions and Difficulty

Next, we analyse the relationship between the two dimensions of Bloom’s Taxonomy, which are conceptually independent. A linear model of difficulty using the levels of both dimensions as factors is significant. Factors CP Understand and KD Factual remain significant as in the individual models, but there are no significant interactions, probably due to sparse data. The raw data still show interesting patterns, though, which we will analyse next.

Table 4 shows the category difficulty means across both Bloom dimensions. Where the table cells are appropriately filled, the mean difficulties reflect the assumptions of the Taxonomy:

CP Remember questions are a lot easier than CP Understand questions (recall the coefficient estimates in Table 2). Within the Remember dimension (the only one to use all three Knowledge levels), mean difficulty rises monotonically in accord with the Knowledge dimension definition.

We now see that the reason for Apply questions overall appearing twice as easy as Remember questions may be the lack of Apply questions using Conceptual and Procedural knowledge. This seems more likely than an effect of noise, since all three Apply questions are at most of difficulty $-0.5$, with an average of $-0.78$, which is clearly on the easy side of the spectrum.

It is also striking that there is an effect of CP level beyond Knowledge dimension for Conceptual, but not Factual questions: The Apply-Factual questions are as difficult on average as the Remember-Factual questions, while the Understand-Conceptual questions are much harder than the Remember-Conceptual questions. Further investigation with a larger data base and more closely standardized Knowledge level annotation would certainly be interesting given this pattern.

In the Knowledge Dimension grand averages, the Taxonomy is clearly mirrored: Questions using Factual knowledge are easier than questions using Conceptual knowledge (this corresponds to the model coefficients shown in Table 3 above). Questions for Procedural knowledge (with an $n$ of just 2) appear overall a little too easy. Keep in mind, though, that the level annotations for the Knowledge dimension must be assumed to be noisy given the low inter-annotator $\kappa$ values.

In sum, both dimensions of Bloom’s Taxonomy taken together categorize the CSSAG questions into four categories of monotonously increasing difficulty in the raw data (ignoring for the moment the Apply-Factual category): Remember-Factual, Remember-Conceptual, Remember-Procedural and Understand-Conceptual. The data confirm that Bloom categories are predictive of question difficulty before testing, allowing teachers and test creators to balance their tests before or even without norming. Vitally, however, the instructional context of the question has to be taken into account for categorization.

4 Answer Variation

Our analyses so far have looked at predicting question difficulty solely from properties of the question (and instructional context), prior to testing. Once the question has been answered, but before grading, another potentially informative predictor of question difficulty becomes available: Answer variation, measured either as the average similarity of student answers among themselves or their average similarity with the reference answer.

We hypothesize a link between answer variation and question difficulty based on the assumption that easy questions (e.g. on the Bloom Remember-Factual levels) have clear-cut answers that many
students know. This should lead to many highly similar student answers (mirroring the reference answer). Difficult questions that require understanding of conceptual knowledge should show higher variation in the phrasing of the correct answer as well as more incorrect answers, leading to higher answer variation both among student answers and with regard to the reference answer.

If such a link indeed exists, then discrepancies between a question’s intended difficulty and its observed answer variation would help identify problematic questions even before grading.

We model average student answer variance through the Greedy String Tiling (GST) similarity measure (Wise, 1996), which ranges between 0 and 1 (where 0 indicates no overlap between the strings – high variation, and 1 indicates perfect overlap – low variation). Comparing the (non-empty) student answers and the reference answer is straightforward. For the average similarity within all non-empty student answers, we use each student answer in turn as the point of comparison since GST is non-symmetric. We use the same corpus as before (see Section 3.1).

Rasch question difficulty (the assumed ground truth) is indeed correlated with the average variation between student and reference answers at Spearman’s $\rho = -0.372, p < 0.05$ and with the average variation of student answers among themselves at Spearman’s $\rho = -0.668, p < 0.001$. For both measures, difficulty is low when answer similarity is high (and therefore, answer variation is low). Perhaps surprisingly, the variation of answers among themselves is a much stronger predictor than variation with regard to the reference answer. This may be because the similarity measure does not account for valid paraphrases (e.g., by technical terms in the reference answer). Relying just on the student answers is more elegant in any case, as no assumptions are made about the quality (or even existence) of the reference answer.

Next, we train a linear model predicting difficulty, just as before, but using student answer variation (SAV) as a factor. Table 5 compares the results for SAV to a model using the Bloom KD and CP text&question levels. We also combine SAV and both Bloom dimensions. We find that all three models significantly predict difficulty. At $n = 30$, there were no significant differences between the models in an ANOVA. We do see some indication of differences between the models in the $R^2$ values, however, which reflect how much of the variance in the variable difficulty the model accounts for. For the combination of the Bloom dimensions, $R^2$ is somewhat higher than for SAV alone, but combining all factors yields another small increase.

We conclude that the Bloom levels, if known, are the best predictors of question difficulty. However, it can be difficult to assign the levels for existing questions if instructional materials are not available. In this case, the amount of within-answer variation for each question can be used to estimate question difficulty before grades and PTT estimates are available, or if the PTT assumptions are not met.

Results from Dueñas et al. (2015) suggest that flat features such as word and length information from the question and reference answer are also useful in predicting difficulty; for them, simplified

<table>
<thead>
<tr>
<th></th>
<th>Factual</th>
<th>Conceptual</th>
<th>Procedural</th>
<th>Grand Avg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remember</td>
<td>$-0.79$</td>
<td>$-0.18$</td>
<td>$0.28$</td>
<td>$-0.39$</td>
</tr>
<tr>
<td>Understand</td>
<td>$-0.78$</td>
<td>$0.63$</td>
<td>$-0.39$</td>
<td>$0.63$</td>
</tr>
<tr>
<td>Apply</td>
<td>$-0.79$</td>
<td>$0.37$</td>
<td>$0.28$</td>
<td>$-0.78$</td>
</tr>
<tr>
<td>Grand Avg</td>
<td>$-0.79$</td>
<td>$0.37$</td>
<td>$0.28$</td>
<td>$-0.78$</td>
</tr>
</tbody>
</table>

Table 4: Cognitive Process text&question and Knowledge dimensions, Rasch difficulty averages (number of questions).

<table>
<thead>
<tr>
<th>Model</th>
<th>Adjusted $R^2$</th>
<th>Model Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>KD + CP text &amp; question</td>
<td>$0.290$</td>
<td>*</td>
</tr>
<tr>
<td>Avg. SAV</td>
<td>$0.246$</td>
<td>*</td>
</tr>
<tr>
<td>SAV + KD + CP text &amp; question</td>
<td>$0.312$</td>
<td>*</td>
</tr>
</tbody>
</table>

Table 5: Difficulty predicted by the Bloom Knowledge dimension (KD) and Cognitive Process (CP) levels and SAV (student answer variation): Linear model $R^2$ values and significances. *: $p < 0.05$. 
taxonomy categories worked better than Bloom categories. Note that they had no information on the instructional materials used and so could not adjust the CP categories (see Section 3.3).

5 Automated Grading: Features and Difficulty

Having looked at difficulty and its predictability from the point of view of test creation in the previous section, we now turn to an analysis of the usefulness of question difficulty information for automated grading.

In Padó (2016), we found that on the corpus level, there are optimal feature combinations for different data sets. Learner corpora of text comprehension questions (lower on the Bloom hierarchy) can be graded well with shallow features close to the string level, while corpora for content assessment of native speakers (containing questions higher on the Bloom hierarchy) require features derived from syntactic and semantic analysis. Following this lead, we investigate the link between question difficulty and optimal feature sets for grading on the question level. We show that question difficulty can indeed be used for question-level model selection (of the optimal feature set). Since question difficulty is often not known at grading time, we also look at Bloom’s Taxonomy levels and SAV as predictors for model selection.

5.1 Automated Grading Model and Features

For reasons of comparability, we use the automated binary grading model from Padó (2016). It consists of a decision tree algorithm that considers features from five feature groups. Table 6 lists them in order of increasing complexity of the linguistic analysis necessary to compute them. We will refer to the NGram as well as the Similarity features (consisting of the Greedy String Tiling, Cosine, and Levenshtein Edit Distance similarity algorithms) as shallow features, because only the character strings of the answers and possibly lemmatization are needed. The deep features are the overlap between student and reference answer in terms of Dependency relations or Lexical Resource Semantics (LRS) components (Richter and Sailer, 2004), as well as the output of the Excitement Open Platform Textual Entailment system (Magnini et al., 2014).

5.2 Method

We train the grading model in the leave-one-question-out setting on the CSSAG corpus (Section 3.1). This means the test questions and answers are completely unseen during training. We do five training and test runs for each question: First with only the NGram features, then adding the Similarity features and so on, until the full feature set is used. We then determine for each question which feature sets yield the best performance. We report per-question prediction accuracy, which ranges between 50 and 88.9%.

5.3 Feature Sets and Model Selection

We find that for 12 out of the 30 available questions, the best performance is only reached using deep features in addition to the shallow features. For the remaining 18 questions, the best performance is already reached using just the NGram or NGram and Similarity features. In seven of these 18 cases, model performance even declines when the deep features are added, for the remaining 11 cases, either feature set yields optimal performance. These results show that there is room for question-level feature optimization.

The short-answer grading model with the full feature set (the best choice for the corpus according to Padó (2016)) reaches an overall accuracy of 73.11%. If we choose the best-performing feature set for each question instead of the full model, overall accuracy increases to 74.35%.

These results indicate that automatic grading can be improved by choosing the best-performing grading model for each question instead of relying on a per-corpus choice. We expect greater improvements with fine-tuned features, because the feature implementations from Padó (2016) were left intentionally vanilla so the results would generalize more easily over the range of corpora used there.

5.4 Model Selection by Difficulty

We continue our analyses with the 19 questions with optimal behaviour for just one feature set. For the other 11 questions, either feature set works well, so they carry limited information for us. To verify that difficulty is indeed related to the optimal feature set for grading, we train a linear model of difficulty using the feature set (deep or shallow) that shows optimal performance for each question. The resulting model significantly ($p < 0.01$) predicts difficulty.
<table>
<thead>
<tr>
<th>Feature Group</th>
<th>Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ngram</td>
<td>Unigram, Bigram, Trigram overlap of student and reference answer</td>
</tr>
<tr>
<td>Similarity</td>
<td>Greedy String Tiling, Cosine, Levenshtein measures</td>
</tr>
<tr>
<td>Dependency</td>
<td>Dependency triple overlap of student and reference answer</td>
</tr>
<tr>
<td>Semantics</td>
<td>LRS component overlap of student and reference answer</td>
</tr>
<tr>
<td>TE</td>
<td>Textual Entailment of reference answer by student answer</td>
</tr>
</tbody>
</table>

Table 6: Overview of the feature set for automated grading

<table>
<thead>
<tr>
<th></th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Baseline</td>
<td>63.2</td>
</tr>
<tr>
<td>Difficulty</td>
<td>78.9</td>
</tr>
</tbody>
</table>

Table 7: Model Selection: Accuracy of predicting the best-performing feature set

We now change tasks and evaluate the usefulness of difficulty for model selection. We evaluate how well the best-performing feature set (shallow or deep) for each question can be predicted by a logistic regression model (\( R^{cv.glm} \)) using difficulty as its only feature.\(^5\) We use leave-one-out cross-validation.

Table 7 shows the classification accuracy of predicting when the shallow feature set will outperform the deep feature set. Using only ground-truth difficulty, the prediction is correct for roughly 80% of the 19 questions. This clearly outperforms the frequency baseline (always predict the deep feature set). Difficulty therefore is very informative with regard to the most useful features for SAG.

5.5 Model Selection: Bloom Levels and SAV

If difficulty estimates are not available, Bloom levels or SAV may still be obtainable. We have shown above that both can be used to predict difficulty. In the case of Bloom levels, we also see a promising pattern in the raw data: There is a clear tendency for questions low on the Bloom hierarchy to be optimally gradable with shallow features, while questions higher on the Bloom hierarchy require deep features. For three out of four Remember-Factual questions (out of the 19 questions with one optimal feature set), optimal grading performance is reached with shallow features. For the five Remember-Conceptual questions, two show optimal performance with shallow and three with deep features. Six out of seven Understand-

Conceptual questions require deep features, and there is one Remember-Procedural question, also optimally graded with deep features. (The two Apply-Factual questions are split between deep and shallow features, in keeping with their estimated difficulty, see Section 3.5).

We therefore use the Bloom levels to train a logistic regression models to predict the optimal feature set, just as above. A second model uses SAV. The left-hand side of Table 8 shows that for our small data set, these factors perform practically at chance level, much below the frequency baseline.\(^6\)

This pessimistic result is not the whole picture. We also evaluate three simple, conservative heuristics based on the Taxonomy, SAV and difficulty, respectively, that do not require training. The results are on the right-hand side of Table 8.

The Bloom heuristic predicts the shallow feature set for all Remember-Factual questions, and the deep feature set otherwise. Its accuracy of 74% clearly outperforms the frequency baseline.

The SAV heuristic predicts the shallow feature set for the 20% of questions with the lowest student answer variation (i.e., highest within-answer similarity). We chose 20% for the boundary based on the observation that there are five Bloom dimension combinations present in the data and the Bloom heuristic assigns the shallow feature set for only one of them. The SAV heuristic performs at the level of the frequency baseline.

The difficulty heuristic predicts the shallow feature set for the easiest 20% of questions. At 84% accuracy, this prediction model even outperforms the linear difficulty model from Section 5.4.

The results underscore the usefulness of difficulty for model selection. In parallel to Sections 3 and 4 above, we find that difficulty can be approximated well by the levels of the Bloom Taxonomy,\(^6\)

Note that our result is strictly speaking an upper bound, since difficulty was originally inferred using all questions.

For the Bloom Taxonomy evaluation, one dimension level was represented only once, so the corresponding data point was unpredictable and was excluded, yielding \( n = 18 \) and a frequency baseline of 61.1.
and to a degree by the variation within student answers, if the levels are not available. While for small data sets such as ours, learning a selection model may not be possible, difficulty and its stand-in measures contain sufficient information to formulate an informative, yet simple heuristic model.

### 6 Discussion and Conclusions

Question difficulty is important in test creation and question analysis (as our discovery and exclusion of an unsuitable question in Section 3.1 demonstrates). We have shown that is also an informative factor in optimizing automated grading: Question difficulty quite accurately predicts which feature set allows best grading performance. This insight allows us to use question difficulty to tailor models to specific questions and optimize SAG performance.

Difficulty, however, can only be estimated after grading, making it impractical to use in many SAG settings. We have shown that difficulty can be approximated by the question’s levels on Bloom’s Taxonomy, a standard tool in education, or, to a somewhat lesser extent, by SAV, the amount of variation present in student answers, measured in string similarity. These approximations are available before testing (Bloom) and grading (SAV).

In this context, we can refine the hypothesis put forward in Padó (2016) that the grading performance variation of different feature sets over different corpora is primarily due to differences in answer variation. Padó (2016) attributes these differences to different student populations (language learners have less ability to paraphrase than native or near-native speakers), which co-varied with elicitation tasks (learner reading understanding versus native content assessment). Our results here zoom in on native-level speakers in content assessment. We found a strong relationship between preferred grading features and question difficulty, while difficulty is partially expressed in answer variation.

The link between Bloom hierarchy levels and difficulty that we found provides more insight: Questions low on the Bloom hierarchy tend to be easier and are optimally graded with shallow features (close to the text level). Questions higher on the Bloom hierarchy require deep features (more extensive syntactic and semantic analysis). This matches the corpus-level results from Padó (2016) (on top of the effect of language ability): The corpora best graded with shallow features were learner corpora of text comprehension questions. Most of these questions are low on the Bloom hierarchy, since they ask the reader to repeat knowledge explicitly presented in the text. On the other hand, content assessment corpora (such as CSSAG) contain more and higher Bloom levels and therefore more questions that require deep processing for grading.

An avenue for future work is the automatic inference of Bloom Taxonomy levels. In addition to facilitating SAG, knowing question difficulty levels without norming would increase the quality of manually created ad-hoc tests as well as automatically generated question sets. The guidelines from Anderson and Krathwohl (2014) suggest that the levels can be inferred from the question wording by Textual Entailment methods. Given the necessary inference steps and the patterns of human annotation consistency for the two dimensions, the Cognitive Process dimension lends itself more to automated assignment than the Knowledge dimension. Finally, we have shown that it is vital to identify cases of recall of instructional materials in level prediction.

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A subset of text comprehension questions from CREG (Meurers et al., 2011) shows 59 Remember and 24 Understand questions out of 83 rated questions.
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